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Abstract:

This deliverable reports on the results in the area of *approximate analysis of quantitative embedded system*, produced in the QUASIMODO project.

Keyword list: Approximation, Metrics, Probability, Analysis

Contents

Abbreviations

AAU: Aalborg University, DK (P01)

ESI/RU: Radboud University Nijmegen, NL (under the auspices of P02)

CNRS: National Center for Scientific Research, FR (P03)

RWTH: RWTH Aachen University, D (P04)

SU: Saarland University, D (P05)

1 Introduction

Quantitative Analysis of Embedded System is facing the need to reason about numerical representations of – in principle real valued – data. In this context, stable and effective approximation algorithms are of central importance. Therefore, approximative analysis methods have been a focus point of the Qasimodo project. This deliverable summarises the results achieved.

The first two sections are devoted to novel results on the analysis of continuous time Markov chains (CTMCs). This is a class of stochastic processes that combines widespread applicability with analytical tractability. Apart from timed automata, CTMCs form the most important base model class for the many quantitative analyses carried out within Quasimodo. Section 4 focusses on a core algorithmic challenge in CTMC analysis, the task of calculating the transient distribution, thus the distribution of probability mass at a given time point t. This is attacked with Krylov subspace methods for stiff CTMCs. Section 3 is devoted to the original CSL model checking problem for CTMCs. The original decidability result for full CSL – with multiple until formulae – was thus far not complemented with an approximative model checking procedure. Our contribution closes this gap, by combining nested transient analyses with automata-based methods.

The subsequent two sections are devoted to novel results for continuous time Markov decision processes (CTMDPs). These are extensions of CTMCs with nondeterminism, which increases the modelling power considerable, and is attractive for abstraction-refinement based analysis approaches (see Deliverable D2.4). Section 2 discusses the problem of time-bounded reachability for CTMDPs in locally uniform CTMDPs and IMCs, and provides approximative techniques to solve this problem. This is a central piece for arriving at model checking procedure for CTMDPs. This problem was open for about ten years. Indeed Section 5 describes the solution to this problem, a full model checking procedure for CSL interpreted on CTMDPs, also supporting reward (or cost) decorations.

Section 6 focusses on a more challenging approximative analysis problem, but for a simpler model class. We consider discrete time Markov chains and decision processes, where certain model parameters are left unspecified. We therefore are facing a parametric model checking problem. Our solution allows us to synthesize parameter ranges for satisfying a given PCTL requirements.

Section 7 discusses a variety of result restablished in a general framework for the analysis of quantitative and qualitative properties of reactive systems, based on a notion of weighted transition systems. Weighted transition systems can be used for specifying the semantics of systems with quantitative and qualitative properties, such as weighted timed automata for example, which feature both weights. We describe how metrics and distances lead to a qunatifiable notion of approximation for weighted transition systems.

In Section 8 we report on an inspiring line of Quasimodo joint work. It offers a natural stochastics semantics of networks of (priced) timed automata. The extension allows for hard real-time properties of timed automata to be refined by performance properties, e.g. in terms of probabilistic guarantees of time- and cost-bounded properties. This enables the application of statistical model checking to efficiently estimate the correctness of model checking problems with a desired level of confidence.

Finally, Section 9 reports on advances in the area of stochastic hybrid systems. This is a very challenging class of models, for which approximative analytical techniques have been rare so far. We describe how for discrete time stochastic hybrid systems, reachability problems can be approximately solved via model checking discrete-time Markov chains.

2 Model Checking Markov Chains using Krylov Subspace Methods: An Experience Report

This work is published in *EPEW 2010* [11].

Participants: Falko Dulat, Joost-Pieter Katoen, Viet Yen Nguyen (RWTH).

Context Stiff continuous time Markov-chains are found in many domains, among which systems biology, where the reaction rates of molecules may vary greatly, and mission critical systems engineering, where failures occur frequently (like sensor glitches) or sporadically (like complete sensor failure). The transient distribution of CTMCs —what is the probability to be in a state at time t?— is a prominent measure of interest, and is fundamental to a range of measures of interest such as time-bounded reachability properties . Its computation is a well-studied topic and a survey of applicable techniques is discussed by De Souza e Silva and Gail. One wide-spread method is Jensen's uniformization which is known for its good numerical stability and is implemented as the default method for transient analysis in various —if not all— Markov analysis tools. Its performance degrades however on stiff models, which, given its many definitions in literature, we simply refer to as the degree of difference between the smallest and largest rates in the CTMC. Other methods like Runge-Kutta solvers require small discretization values on stiff models, thereby suffering from similar performance problems. On top of these problems, potential numerical instability, not uncommon with stiff models, needs to be dealt with as well.

Contribution In this work, we reintroduce a Krylov-based method for computing the transient distribution of a CTMC. It is briefly mentioned in Moler and Van Loan's discourse on 19 methods for the matrix exponential as a novel 20^{th} method and in De Souza e Silva and Gail's survey as a possible method for computing the transient distribution of a CTMC. Despite these references and their success for many matrix-related computations in different fields of science and engineering, Krylov-based methods received scant attention in the field of probabilistic analysis. We believe this is due to three reasons, namely (i) to our knowledge, experiments with a Krylov-based method have been only conducted on small academic examples or without regard to stiffness versus non-stiffness (ii) due to the lack of the former, nobody has identified the class of CTMCs for which Krylov-based methods excel and (iii) the good applicability of Krylov-based methods to the transient have, to our knowledge, not been explained theoretically. This report addresses, among things, these issues:

1. We apply a Krylov-based method for computing the transient distribution of CTMCs to model check time-bounded reachability properties expressed in Continuous Stochastic Logic (CSL).

2. We extensively compare the implemented Krylov-based method to the existing uniformization-based method on five case studies from the literature comprising various application domains.

3. We identify that computing the transient distribution is (much) faster with Krylov-subspace methods for a particular class of models, namely stiff CTMCs.

4. We provide an explanation of the good approximation properties of the Krylov-based matrix exponential using Schwerdtfeger's formula [23].

Perspective The overall result is to reintroduce Krylov-based methods to the probabilistic community as the preferable method for analysing *stiff* CTMCs as substantiated by means of an extensive experience report.

3 Automata-based CSL model checking for Markov Chains

This work is about to be published in *ICALP 2011* [26].

Participants: David N. Jansen (ESI/RU), Holger Hermans (SU),
Flemming Nielson, Lijun Zhang (DTU Informatics, DK).

Context For continuous-time Markov chains, the model-checking problem w. r. t. continuous-time stochastic logic (CSL) has been introduced and shown to be decidable by Aziz, Sanwal, Singhal and Brayton in 1996 [2]. The presented decision procedure, however, has exponential complexity. In 2000, Baier *et al.* [3] presented an *approximate model checking algorithm* for a sublogic of PCTL. This algorithm is based on transient probability analysis for CTMCs. More precisely, it was shown that $\Pr_s(\varphi)$ can be approximated, up to a priori given precision ε, by a sum of transient probabilities in the CTMCs. Their algorithm then led to further development of approximation algorithms for infinite CTMCs and abstraction techniques. More importantly, several tools support approximate model checking, including PRISM [18] and MRMC [20].

Efficient model checking of full CSL with multiple until formulae (of the form $\mathcal{P}_{\geq p}(f_1\ U_{I_1}\ f_2\ U_{I_2}\ \ldots\ U_{I_{k-1}}\ f_k)$) is an open problem. This problem is gaining importance e. g. in the field of system biology, where one is interested in oscillatory behaviour of CTMCs [5, 24]. More precisely, if one intends to quantify the probability mass oscillating between high, medium and low concentrations (or numbers) of some species, a formula like $high\ U_{I_1}\ medium\ U_{I_2}\ low\ U_{I_3}\ medium\ U_{I_4}\ high$ is needed, but this is not at hand with the current state of the art.

Contribution In this paper we propose an approximate algorithm for checking CSL with multiple until formulae. We introduce a subclass of *stratified CTMCs*, on which the approximation of $\Pr_s(\varphi)$ can be obtained by efficient transient analysis. Briefly, a CTMC is stratified with respect to $\varphi = f_1\ U_{I_1}\ f_2\ U_{I_2}\ \ldots\ U_{I_{k-1}}\ f_k$, if the transitions of the CTMC respect some *order* given by the f_i. This specific order makes it possible to express $\Pr_s(\varphi)$ recursively: more precisely, it is the product of a transient vector and $\Pr_{s'}(\varphi')$, where φ' is a subformula of φ. Stratified CTMCs are the key element for our analysis: In a stratified CTMC, the problem reduces to a transient analysis. We extend the well-known result [3] for the case of binary until. Efficient implementations using *uniformization* [15] exist.

For a general CTMC, we present a measure-preserving transformation to a stratified CTMC. Our reduction is described using a *deterministic finite automaton* (DFA) over the alphabet $2^{\{f_1,\ldots,f_k\}}$. The DFA accepts the finite word $w = w_1 w_2 \ldots w_n$ if and only if the corresponding set of time-abstract paths in the CTMC contributes to $\Pr_s(\varphi)$, i. e., it respects the order of the f_i. The transformation does not require to construct the full DFA, but only the product of the CTMC and the DFA. We show that the product is a stratified CTMC, and moreover, the measure $\Pr_s(\varphi)$ is preserved. This product can be constructed in linear time and space.

Recently, the decision algorithm by Aziz *et al.* was shown to produce erroneous results on some non-stratified CTMCs [19]. Still, their algorithm is correct on stratified CTMCs. As an

additional contribution, our measure-preserving transformation ensures the decidability of CSL model checking for general CTMCs.

Perspective Our method will be useful as the centrepiece of a full CSL model checker equipped with multiple until formulae.

4 Time-Bounded Reachability Probabilities in Continuous-Time Markov Decision Processes

This work is published in *QEST 2010* [22].

Participants: Martin R. Neuhäußer (RWTH),
Lijun Zhang (DTU Informatics, DK).

Context Continuous-time Markov decision processes (CTMDPs) are a stochastic model which allows for nondeterminism between transitions whose delay is governed by negative exponential distributions. As such, CTMDPs extend continuous-time Markov chains (CTMCs) with nondeterministic choices and discrete-time Markov decision processes (MDPs) with exponentially distributed delays.

As CTMDPs in general exhibit nondeterminism, their induced stochastic process is not uniquely determined. Therefore, we follow the MDP approach and define schedulers that resolve the nondeterministic choices: Depending on the trajectory that led into the current state, a scheduler returns a probability distribution over the available actions and thereby resolves the action-nondeterminism in that state. Accordingly, the stochastic behaviour of a CTMDP is described by upper and lower probability bounds induced by a given—usually uncountable—class of schedulers.

In general, the sojourn time distribution of the current state depends on the action that is chosen by the scheduler. This dependency requires the scheduler to decide early, that is, when entering the current state. Accordingly, we refer to such schedulers as early schedulers. However, locally uniform CTMDPs—which share the property that their states' residence time distributions do not depend on the scheduler's choice—allow for even more powerful schedulers: As shown in Quasimodo Deliverable D2.2 (Section 1.3) local uniformity allows us to delay the scheduling decision until the current state is left; the resulting late schedulers, which are well-defined only for locally uniform CTMDPs, perform at least as good as any early scheduler and generally induce strictly better probability bounds.

Contribution By first restricting ourselves to locally uniform CTMDPs, the time-bounded reachability problem can be solved in that we compute the maximum probability to reach a set G of goal states within a given time bound z under all late schedulers. More precisely, we characterise the maximum time-bounded reachability probability as the least fixed point of a higher-order operator which involves integration over the time domain. Exploiting this result, we prove that for time-bounded reachability, it suffices to consider late total time positional deterministic schedulers (TTPD) which base their decision only on the elapsed time and on the current state. This allows us to reduce the problem of computing time-bounded reachability probabilities in locally uniform CTMDPs to the problem of computing step-bounded reachability probabilities in discrete-time MDPs.

Specifically, we show how to approximate the behaviour of the locally uniform CTMDP up to an a priori specified error bound $\varepsilon > 0$ by defining its discretised MDP such that its maxi-

mum step-bounded reachability probability coincides (up to ε) with the maximum time-bounded reachability probability of the underlying locally uniform CTMDP. Computing the maximum step-bounded reachability probability in MDPs is a well-studied problem and can be done efficiently, e.g. by value iteration algorithms. Furthermore, a small extension of the value iteration algorithm allows us to automatically synthesise the ε-optimal scheduler which induces the maximum time-bounded reachability probability.

Subsequently, we turn our attention to the problem of computing time-bounded reachability probabilities for general CTMDPs. In this setting, late schedulers are not applicable. Hence, we resort to early schedulers and introduce a measure preserving transformation from arbitrary CTMDPs to interactive Markov chains (IMCs). This allows for the exploitation of earlier results obtained within the Quasimodo project (see Deliverable D2.1, Section 1) to solve the time-bounded reachability problem for IMCs. Hence, the maximum (and minimum) time-bounded reachability probabilities for early schedulers and general CTMDPs can be computed by analysing the CTMDPs' induced IMCs. In both cases, the complexity is in $O(m{\cdot}(\lambda{\cdot}z)^2/\varepsilon)$, where m denotes the size of the input model, λ is its maximal exit rate and z the given time bound.

Perspective All results holds for maximum time-bounded reachability probabilities and minimum time-bounded reachability probability. The reachability analysis is the key ingredient to enable approximate model checking of CTMDPs with respect to logics like CSL. This tangible result can thus be considered as the corner stone for a full CSL model-checking algorithm for CTMDPs.

5 Approximate CSL Model Checking for Continuous-Time Markov Decision Processes

This work is published in *CAV 2011* [8].

Participants: Ernst Moritz Hahn, Holger Hermanns (SU),
Peter Buchholz (TU Dortmund, Germany),
Lijun Zhang (DTU Informatics, DK).

Context The approximation of performance and dependability properties of continuous-time Markov Decision processes (CTMDPs) is much more involved than analyses of their discrete-time counterpart. This is because for computing properties like the probability to reach a set of states within a given time bound there are much more dimensions for the classes for schedulers to consider:

- time-dependence or time-independence,
- late or early decisions,
- history-dependent or stationary decisions

Contribution We have developed an analysis method that allows us to efficiently model check full CSL formulae for CTMDPs. The most complicated part is the analysis of the time-bounded until operator. We consider the scheduler class which is known to be the most powerful one for this property class. We explained how to use previously existing technique to build a full model checking procedure for this logic and model class. A preliminary implementation has been implemented in the probabilistic model checker MRMC [21], and has been successfully applied on several case studies.

Perspective Several extensions appear promising. First, one can think of extending existing techniques to approximate properties in extended model classes, like Markov automata [12]. We also want to provide estimates on how far values computed by the algorithms of [8] and [4] may vary, depending on the structures of a CTMDP.

6 Synthesis for Parametric Markov Models

This work is published in *NFM 2011* [16].

Participants: Ernst Moritz Hahn, Holger Hermanns (SU),
Tingting Han, Björn Wachter (University of Oxford, UK),
Lijun Zhang (DTU Informatics, DK).

Context When considering a given probabilistic Markov model, we usually assume that we are fully aware of probability distributions (or rates) occurring at any place. This may however not always be the case. We might have certain knowledge about the general structure of the model and the range of probabilities but now their exact values. We might thus consider a parametric model, where probabilities are no fixed, but specified as functions over a given set of model parameter. Given such a parametric model, questions of interest are now for which parameter values a certain property holds, or which are the optimal parameters with respect to a property.

Contribution In [16] we have extended our existing tool PARAM [17] (see Deliverable D2.3, Section 6) to perform parameter synthesis for PCTL in parametric Markov decision processes. Here, we were assuming that we are given a PCTL formula and a parametric Markov decision process. We were able to synthesise the parameter regions which fulfil this specification. A region is a hyper-rectangle in the dimension of the model parameters, representing the concrete models resulting from instantiations of the parameters with values in this region. We have developed a preliminary implementation which we applied successfully on a case study. The PCTL formulae we can handle also comprehend an extension to allow reasoning about the expected accumulated reward in a Markov reward model until a given set of states is reached.

Perspective In the future, we are planning to handle other properties than just standard PCTL, for instance also the reward-bounded until properties. We are also planning to use different representations of parameter regions than hyper-rectangles.

7 Metrics for Weighted Systems

This collection of work has been published in [25, 13, 14].

Participants: Uli Fahrenberg, Kim G. Larsen and Claus Thrane (AAU)
Patricia Bouyer, Nicolas Markey, Ocan Sankur (CNRS).

Context The motivation of this line of contributions follow the Embedded Systems challenge put forward by Henzinger and Sifakis: for embedded systems monitoring and controlling a continuous environment, the challenge is to replace the absolute (boolean) notions of program correctness as classically applied in Computer Science with a continuum of (real-valued) degrees of adequacy. E.g. rather than declaring a system model to be correct or incorrect with respect to a logical property we will measure the degree by which a the system can be seen to satisfy the property, and rather than declaring two system models to be equivalent or nonequivalent we will measure the distance between to systems.

Contribution In [25] we present a general framework for the analysis of quantitative and qualitative properties of reactive systems, based on a notion of weighted transition systems. Weighted transition systems can be used for specifying the semantics of systems with quantitative and qualitative properties, such as weighted timed automata for example, which feature both weights and time. We introduce and analyse three different types of distances on weighted transition systems, but note that other interesting types may be treated in a similar manner. The three types are *pointwise* distance, which measures the largest individual difference between systems, *accumulated* distance, which measures the sum of (absolute) differences accumulated during executions of the systems, and *maximum-lead* distance, which measures the largest distance between accumulated differences occurring during executions of the systems.

All three kinds of distances are defined and analyzed both in a linear setting, i.e. extending the standard notion of trace inclusion, and in a branching version, generalising the notion of simulation. We find that the usual relation between simulation and trace inclusion generalises to our quantitative setting. We apply our quantitative framework to implementation verification for weighted timed automata, and we collect evidence that the standard result on undecidability of timed language inclusion for timed automata can be lifted to our quantitative setting, and that on the other hand (and again generalising standard results), simulation distances are computable for weighted timed automata.

In [14] we provide general framework for reasoning about distances between transition systems with arbitrary quantitative information. Taking as starting point an arbitrary distance on system traces, we show how this leads to natural definitions of a linear and a branching distance on states of such a transition system. We show that our framework generalises and unifies a large variety of previously considered system distances, and we develop some general properties of our distances. We also show that if the trace distance admits a recursive characterisation, then the corresponding branching distance can be obtained as a least fixed point to a similar recursive

characterisation. The central tool in our work is a theory of infinite path-building games with quantitative objectives.

In [13] we a quantitative interpretation of CTL is given with respect to weighted transition systems, giving a real-valued distance, describing the degree of satisfaction. In particular it is proved that there is a close correspondence between the distance between two systems and the degree by which they satisfy certain logical properties, providing a quantitative generalization of classical characterisation theorems linking behavioural equivalences and preorders with temporal logics.

In [6] complexity results and axiomatic proof systems for simulation distance between finite weighted Kripke structures are given. Finally, the [7] paper links the notion of metrics on behaviours to the notion of robustness studied for timed automata – robustness in the sense that (arbitrary) small clock drifts or inaccuracies in clock values do not affect reachability properties in the limit. More precisely, a construction is given which for any timed automaton A and any desired precision $\epsilon > 0$ produces timed automaton A_ϵ being robust and ϵ-close to A, and hence preserving, up to the error ϵ, all properties expressed in (a quantitative) extension of CTL.

Perspective Metrics for weighted transition systems have proven to be a very adequate, rich and powerful way to relax classical program correctness notions towards a continuum of values.

8 Statistical Model Checking for Timed Systems

This work is published in *CAV 2011* [10].

Participants: Alexandre David, Kim G. Larsen, Marius Mikucionis (AAU),
Axel Legay, Rennes (IRISA/INRIA, Rennes, F.),
Zheng Wang (East China Normal University, Shyanghai, China).

Contribution In [9] we offer a natural stochastics semantics of (networks of priced) timed automata based on races between components providing the basis for an interpretation of probabilistic weighted CTL. In particular the extension allows for hard real-time properties of timed automata to be refined by performance properties, e.g. in terms of probabilistic guarantees of time- and cost-bounded properties. Moreover, the stochastic interpretation enable the application of Statistical Model Checking (SMC) to efficiently estimate the correctness of (non-nested) PCTL model checking problems with a desired level of confidence, based on a number of independent runs of the model. In addition to applying classical SMC algorithms, we also offer an extension that allows to efficiently compare performance properties of NPTAs in a parametric setting.

In [10] we provide an implementation of the above work within the Uppaal tool set. One of the major differences with classical Uppaal is the introduction of a new user interface that allows to specify CSTAs with respect to a stochastic semantic; such semantic is naturally needed to apply SMC. Another contribution is the implementation of several versions of the sequential hypothesis testing algorithm of Wald. Contrary to other implementations of SMC, we also consider those tests that can compare two probabilities without computing them.

Perspective Our tool comes with a wide range of functionalities that allows the user to visualise the results on the form of probability distributions, evolution of the number of runs with timed bounds, computation of expected values, etc.

9 Approximate Model Checking of Stochastic Hybrid Systems

This work is published in the *European Journal of Control, 2010* [1].

Participants: Joost-Pieter Katoen (RWTH)
Alessandro Abate (TU Delft, NL)
John Lygeros (ETH Zurich, CH)
Maria Prandini (Politecnico di Milano, It).

Context Stochastic hybrid systems are a broad and widely applicable class of dynamical systems that involve the interaction of discrete, continuous, and probabilistic dynamics. Because of their generality, stochastic hybrid systems have found applications in many areas, including telecommunication networks, manufacturing systems, transportation, and biological systems. The importance of stochastic hybrid systems in applications has motivated a significant research effort into the foundations, analysis and control methods for this class of systems. Among the different problems addressed in this effort, of particular interest are the problems of reachability and invariance, i.e., the characterisation of the probability that the state of a stochastic hybrid system will reach (or, respectively, remain) in a specific region of the state space.

Many of the methods proposed in the area of stochastic hybrid systems for achieving this objective are based on numerical computations. These involve either imposing a grid on the state space, thus turning an infinite state problem into an approximate finite state one, or carrying out Monte-Carlo simulations to obtain empirical estimates of quantities such as expected values of reach probabilities. An alternative approach to the problem of verification of stochastic hybrid systems is based on satisfiability modulo theory. Even though computational tools based on numerical methods typically come with explicit approximation guarantees, their versatility and their computational requirements often limit their applicability to practical problems.

Contribution To address a wider range of problems one would ideally like to combine numerical approximation with symbolic computation techniques that can be used to test a wider range of properties and that have been optimised for computational efficiency. Model checking is an interesting class of methods in this context. Model checking methods provide the means to algorithmically check whether a system satisfies a wide range of properties related to its evolution in time. In the context of reachability, model checking typically involves constructing forward/backward reachable sets based on a model of the system. More generally, model checkers can be employed to verify whether a model of the system satisfies various properties expressed in an appropriate temporal logic.

A key difficulty in deploying model checking methods to hybrid systems is our ability to "compute" with sets, i.e., to represent sets of states and propagate them through the system dynamics. For finite state systems this is not an issue, at least conceptually. Storing and manipulating sets of states can be done either naively by enumeration, or in a more sophisticated way by using efficient representations such as binary decision diagrams; as a consequence, model checking tools for deterministic, discrete time, finite state systems have been available for many

years and have been successfully used in numerous applications. For systems whose state involves infinite or uncountable components it is sometimes possible to obtain an equivalent finite state representation on which finite state model checking methods can be applied.

Here, we take a first step toward combining numerical methods for approximate computation in stochastic hybrid systems with model checking methods developed to test temporal logic properties for finite state Markov chains.

Perspective For the time being we concentrate on discrete time stochastic hybrid systems and finite time invariance specifications; current work focuses on extending the results to a wider range of properties of interest coded in the Probabilistic Computational Tree Logic (PCTL). The main idea is simple: given a stochastic hybrid system, we use numerical tools to generate a finite state Markov chain, together with guarantees on the level of approximation introduced in the process. The properties of the Markov chain (in our case the probability of remaining in a certain region of the state space) are then analyzed using a model checker. The result is combined with the approximation guarantees to provide an overall guarantee about the probability of satisfying the original property of interest for the stochastic hybrid system.

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